Discrete Events and Max-Algebra

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I have been very pleased to observe the recent blossoming of interest, within Holland, in the application of max-algebra to the study of discrete-event systems, as a good deal of my own work in this area was done when I held an appointment at Twente.

It is, however, not true that a 'connection with dynamic problems ... seems to be new', as J.M. Schumacher writes in a recent edition of CWI Quarterly [7]. A glance at the early work of Giffler [6], or myself [1], dating back to the 1960's will show formulations like those labelled (1) to (8) by Schumacher. The use of linear max-algebra to describe the evolution through time of discrete-state systems has *always* been to one of the principal motivations in developing such a theory, as is explained in the introductory section of [2] (1979).

It is, therefore, a little wide of the mark to describe [2] as essentially a theory of linear equations. Linear *algebra*, yes. But that embraces also the matters very relevant to, and explicitly motivated by, the need to analyse dynamic behaviour.

It has been the fate of this subject to be re-discovered independently several times over, and the literature is extensive. The book by Zimmermann [8] gives an entree to early work by researchers in France, Germany, Austria, Czechoslovakia and elsewhere on dioid theory generally. In relation to the particular topic of discrete-event systems, Schumacher cites work by Olsder (1986) and by Moeller (1986) on Cayley-Hamilton and the characteristic equation, but other work, e.g. [5] (1983) is also relevant, and relates the linear-algebraic theory of polynomials and rational functions over max-algebra [3].

Finally, in discussing the long-term behaviour of dynamic systems, some consideration of convergence is necessary, such as is undertaken in e.g. [4].

Readers familiar with the children's classic 'Wind in the Willows' by Kenneth Grahame will recall how Mr. Toad compiled a programme of entertainment in which all the best contributions were by himself. I am conscious of being almost as fat-headed in citing so much of my own work. But the range of journals carrying articles relevant to this subject is so great that newcomers to the field, inspired by excellent articles such as Schumacher's to undertake related research, can easily be unaware of some of the things which have already been done.

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